Velocity filtering of seismic reflection data

P. A. F. Christie,* t V. J. Hughes* & B. L. N. Kennett**

The term 'velocity filtering' may be applied to any process which seeks to separate coherent energy incident upon a seismic array by using the apparent velocity of propagation across the array as a discriminant. The pie-slice process was carried out using a limited multitrace operator in the time-distance (t-x) domain. In order to economise on the convolutions involved, a typical operator consisted of 21 time-points in length, covered 12 traces and resulted in a rejection level about 20 dB down with a less than ideal response function.

While the pie-slice or fan filter may be suitable for stacked profiles, we wish to propose the velocity filtering of unstacked data in the frequency-wavenumber (ω-k) domain as a more suitable alternative. Providing certain precautions are taken in handling the dataset, several advantages of this procedure are obtained:

- the number of output traces is equal to the number of input traces,
- the ideal filter response is more closely achieved,
- no filter coefficients need be computed beforehand,
- implementation of the filter is by straightforward multiplication,
- interactive examination of the data in the (ω-k) domain permits greater flexibility in the design of the filter response,
- since the eye follows group velocity in the (t-x) domain, the presentation in (ω-k) space allows a better estimate to be made of phase velocities and the degree of aliasing,
- other wavefield operations may be performed on the data while in (ω-k) space.

Velocity filtering in the (ω-k) domain is particularly important for the high-resolution, single geophone data recorded in the United Kingdom by the National Coal Board and examples of such data are presented in the paper.

One of the objects of successful filter design is to achieve maximum separation between those signals which are to be retained and those which are to be suppressed. This may be realised if we can find a suitable representation or domain in which the data can be manipulated. The mute and low pass filter are simple examples of one-dimensional filters using time of arrival and frequency content respectively as discriminants. The two-dimensional velocity filter, as its name suggests, uses the apparent (horizontal) velocity of coherent energy across a seismic aperture to discriminate between wanted and unwanted signals. As such, the concept is not new and indeed the pie-slice or fan filter has been in use for many years now. However, until recently, the implementation of the velocity filter has been performed in the time-distance or (t-x) domain. The advent of truly fast, fast Fourier transforms has enabled the advantages of working in the more natural (ω-k) domain to be realised. The purposes of this paper are to discuss the implementation of the (ω-k) algorithm and to point out its advantages with specific regard to the type of high-resolution data recorded by the National Coal Board in the United Kingdom.

The Time–Distance (t-x) Algorithm

The derivation of the least-squares equations governing the computation of the (t-x) velocity filter operator have been discussed fully in a number of papers including Fail and Grau (1963), Embree, Burg and Backus (1963), Wiggins (1966) and Treitel, Shanks and Frasier (1967) and need not be described in detail here. In brief, the desired filter transfer function $H(\omega,k)$ is realised in the (ω-k) plane and in its simplest form consists of a pass-band and two stop bands separated by lines passing through the origin with slopes of ±V where V is the chosen cut-off velocity (Fig. 1), i.e.

$$H(\omega,k) = \begin{cases} 1, & |\omega/k| > V \\ 0, & |\omega/k| < V. \end{cases}$$

Fig. 1. The simplest form of the velocity filter transfer function designed to discriminate against events with an apparent speed across the seismic aperture less than V, the slope of the diagonal lines.

Negative frequencies are not shown. Since seismogram time series are real their Fourier transforms have Hermitian symmetry about zero frequency.
The (ω−k) plane is bounded by the Nyquist wavenumber \( k_n \) and the Nyquist frequency \( f_n \), which are determined by the spatial sampling \( \Delta x \) and temporal sampling \( \Delta t \) by the relations

\[
\begin{align*}
k_n &= \frac{1}{2\Delta x} \\
f_n &= \frac{1}{2\Delta t}.
\end{align*}
\]

It should be emphasised here that the wavenumber, \( k \), is in fact the horizontal wavenumber and not the magnitude of the total wavenumber vector \( k \). Thus, \( V \) is the horizontal velocity across the seismic array equal to the reciprocal moveout. We use the notation (ω−k) to denote the frequency–wavenumber domain since \( \omega \) and \( k \) are the Fourier conjugates of \( t \) and \( x \) respectively. However, plots presented here of the (ω−k) domain are scaled in \( f = \omega/2\pi \) and \( k = k/2\pi \) since these units are more natural for practical use. Again, for convenience, we have used \( k \) for the horizontal wavenumber axis on diagrams.

The filter transfer function is then two-dimensionally discrete Fourier transformed into the (t−x) domain, where it is usually represented by a rectangular matrix of filter coefficients, the rows of which are discrete time samples \( \Delta t \) apart and the columns are separated by the trace spacing \( \Delta x \). The filter is implemented by convolving the columns of the matrix with the seismogram time series and then summing across the rows in order to produce a single output trace, usually referenced to the centre position of the matrix (Fig. 2). The matrix is then stepped one trace or column and the process is repeated. Considerations of economy, selectivity and rejection level determine the size of the matrix but it is intuitively obvious from the uncertainty principle that truncation of the matrix, either in time or distance must necessarily result in a non-ideal realisation of the filter transfer function in the (ω−k) domain.

An example of the true response function of a truncated filter matrix may be seen in Fig. 3, taken from the paper by Embree et al. (1963). This shows the contours of the response in the (ω−k) plane of a 12 trace, 21 point (t−x) operator. The scales along the frequency and wavenumber axes are in fractions of the sampling frequency using the assumption that the data are band limited such that \( f_n = V k_n \) where \( V \) is the cut-off velocity, \( k_n \) is the spatial Nyquist and \( f_n \) is the temporal Nyquist, i.e. \( V = \Delta x/\Delta t \), the ratio of the sample intervals in distance and in time. After Embree et al. (1963).

![Fig. 3. Contours in the (ω−k) domain of the response of a 12 trace, 21 point (t−x) operator. The scales along the frequency and wavenumber axes are in fractions of the sampling frequency using the assumption that the data are band limited such that \( f_n = V k_n \) where \( V \) is the cut-off velocity, \( k_n \) is the spatial Nyquist and \( f_n \) is the temporal Nyquist, i.e. \( V = \Delta x/\Delta t \), the ratio of the sample intervals in distance and in time.](image)
be reduced substantially. Treitel et al. (1967) have shown that by factorising the least-squares algorithm and time shifting, it is possible to reduce the number of convolutions to one per output trace and, by making this operation recursive, the length of the convolutional operator can be shortened to about a third of the non-recursive length.

An arbitrarily large number of traces is combined to produce each output trace and, consequently, the output dataset is reduced in size by the number of traces in the operator unless the input dataset is padded with zero traces. In the case of velocity filtering before stack, which is described below, this could reduce the fold of cover.

The greatest disadvantage of the (t-x) operator, however, lies in the fact that it is implemented without knowledge of the behaviour of the data in the (ω-k) plane. The fan filter is essentially a phase velocity filter and it is to the Fourier components of a particular wavelet that the filter is applied, not to the wavelet envelope. Quite simply, in the time domain, the eye tends to follow group velocity which may be significantly different from phase velocity in the case of dispersed surface waves such as ground roll. It might even happen for a higher mode of ground roll that the phase velocities of its Fourier components do not separate from those of the desired reflected events.

When both data and operator are of necessity discrete, further problems arise due to aliasing. While the individual seismograms may be sampled sufficiently well in time to avoid frequency aliasing, the generally coarse sampling in distance introduces spatial or velocity aliasing since the degree of aliasing is a function of the moveout of a given frequency component. If the moveout per trace is greater than half the period of the maximum frequency component of the wavelet being considered, then the event is velocity aliased (Fig. 4). Again, examination of the data in the time domain may not reveal the extent of velocity aliasing in the dataset.

**U.K. National Coal Board data**

The National Coal Board in the United Kingdom is currently acquiring high-resolution data in an effort to

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**Fig. 4.** The aliasing conditions applied to a monochromatic Fourier component. \( Δt = \text{moveout per trace}, Δx = \text{trace separation}, T_o = \text{period of frequency component}.\) After Ziolkowski (1979).

**Fig. 5.** A set of National Coal Board monitor records displaying the fundamental mode and higher modes of ground roll. Low-cut filtering has not been applied and time-variant gain has been used in plotting the records. After Ziolkowski and Lerwill (1979).
map faults in coal bearing sequences with throws of the order of 5 m (Ziolkowski and Lerwill 1979, Ziolkowski 1979). The data so obtained are usually sampled at 1 or 2 msec in time and at about 5 m in distance. Both the spatial sampling and the desire for faithfully reproducing high temporal frequencies to aid resolution militate against the use of geophone patterns to attenuate ground roll, which is consequently a problem (Fig. 5). Instead, the placing of shots below the weathered layer and the use of small charges to pump less energy into the ground roll spectral range are practices designed to reduce the generation of ground roll. However, ground roll cannot always be avoided in this way and, since the dynamic range of the recording system often cannot resolve the low amplitude reflection on top of the large amplitude ground roll, the ground roll is attenuated by the application in the field of low-cut filtering from about 30 Hz, which of course decreases the signal bandwidth. Low-cut filtering is usually effective in removing the fundamental mode of the ground roll but the higher modes often remain in evidence (Fig. 6). At this point, it is clear that velocity filtering before stack is required using an algorithm which is effective over all the traces in the stack and does not reduce the fold of the cover. These considerations suggest that the velocity filter should be implemented in the \((\omega-k)\) plane after the double Fourier transformation of the data.

**Dataset Organisation**

The shotpoint gather illustrated in Fig. 6 was chosen as a test dataset. Since the successful operation of the velocity filter critically depends upon the good coherency of both signal and ordered noise, a shot point gather was used instead of a common mid-point gather. This particular gather has the shot point close to one end of the profile and although it has been plotted with time-varying gain, all the processing to be described has been performed upon the data before the application of gain at the plotting stage. All plotting parameters for the dataset have been held constant for subsequent plots.

**Periodicity of the Discrete Fourier Transform**

Since the dataset is finite in both the temporal and spatial dimensions, the operation of the two-dimensional discrete Fourier transform assumes periodicity of the dataset in both dimensions so that if the dataset appears schematically in Fig. 7a, what the transform sees is shown in Fig. 7b.

The periodicity in time is not a great problem since the seismic trace starts with zeros and, by tapering the tail to zero with a \(\cos^2\) function over about 20 time points, the zone of wrap-round can be made smooth and continuous. Rather more serious is the discontinuity at the junction of the panels in the \(x\)-direction. Here the Fourier representation of the traces will make a bad approximation to the actual phase behaviour, so that the traces adjacent to the discontinuity will be poorly represented. It should be emphasised that the faulty representation resulting from the discontinuity is no problem so long as we do nothing to the dataset while...
Fourier Transforming the Data

As the seismic traces are read from tape, the Fourier transform from time to frequency is performed and the Fourier coefficients are stored in the notional columns of a rectangular, complex matrix in core. For this particular dataset, 420 time-points were used, padded out with zeros to a transform of 512 time points in length. The edges of the data array are then tapered in magnitude according to a cos² function over the last two traces and blank columns are added to the matrix to raise the number of columns to a convenient power of 2. Fourier transforms are then taken over the rows of the matrix and the resulting coefficients are returned to the array which now contains the two-dimensional discrete transform of the original (t-x) data. At this point, it is convenient to examine the amplitude distribution of the data within the (ω-k) plane, which may be displayed in both projective and contour forms. Examples of both plots are shown in Fig. 8.

Each plot is self-scaled to the maximum amplitude in the (ω-k) plane and, in the case of the contour plot, contours are drawn at intervals of one-tenth of the peak amplitude. The projective plot indicates very effectively the topography of the (ω-k) plane and it is complemented by the contour plot which is more precise in its location of events. The positive half plane only is displayed from 0 to 150 Hz in frequency and bounded by the positive and negative Nyquist wavenumbers. Since the seismograms are real, the negative frequency half of the (ω-k) plane is simply the complex conjugate of the positive half plane, and so need not be displayed. The projective plot clearly shows the effect of the 30 Hz low-cut filter applied in the field to suppress the fundamental mode of ground roll. All energy dies out at about 130 Hz, which is well removed from the Nyquist frequency of 250 Hz. The temporal Nyquist frequency, fₙ, is determined by the sampling in time and is relatively cheap to vary in the field. However, the spatial Nyquist wavenumber, kₑ, is fixed by the field configuration, over which there is little control except in the station interval. In terms of wavenumber, the containment of energy within the first Brillouin zone (i.e. within the interval ±kₑ) is not quite so good, as may be seen by the non-zero edges to the projective plot but since they fall between the one-tenth and one-fifth contours this degree of aliasing is probably acceptable.

The strongest event A in both plots lies in the middle with its major axis inclined to the positive quadrant side of the frequency axis as shown on the contour plot. This represents energy reflected from shallowly dipping beds with small positive moveout and therefore high horizontal phase velocity. From the central lobe of these arrivals a spur of high frequency energy, B can be seen which is separated from the event A by a narrow trough. This event is interpreted as being the first arrivals which have a higher frequency content than the deeper reflec-
Fig. 8. Projective (a) and contour (b) plots of the amplitude distribution in the \((\omega - k)\) plane after the two-dimensional fast Fourier transform of the dataset illustrated in Fig. 6. Event A—sub-horizontal reflections; event B—first arrivals; events C and D—ground roll.

Noting that lines along the crests of the arms do not intersect the origin, indicating that the group velocity \((d\omega/dk)\) is significantly less than the phase velocity \((\omega/k)\), which is characteristic of the dispersive nature of higher mode Rayleigh wave propagation in a stratified medium. In this case, an examination of the data in the time domain would probably result in the choice of a cut-off velocity too low for effective suppression of the ground roll. Again it is important to stress that while the two-dimensional FFT is a ‘phase analyser’, the identification of events in the \((\omega, k)\) domain has been performed principally on the basis of trend, or group velocity, which is the velocity observed in the time domain.

Implementation of the Velocity Filter

The examination of the data in the \((\omega - k)\) domain has revealed:

— that the degree of aliasing in this configuration is tolerable,
— that the ground roll does indeed separate from the desired signal,
— that the choice of the cut-off velocity must be significantly higher than the group velocity of the ground roll indicated in the time domain.
A cut-off velocity of about 1.5 km sec\(^{-1}\) passes conveniently up the valley between the events B and C and this was indeed chosen for the filter response. A straight line cut is particularly easy to implement since all points below the line are set to zero, resulting in infinite attenuation. Grid points above and close to the line are tapered with a \(\cos^2\) window over a given number of points perpendicular to the line of the cut. All points above the line and outside the zone of smoothing are passed by multiplying by unity. An example of the filter response function is shown in Fig. 9 where the smoothing has been made over 8 points. Each star is simply the equivalent time-distance operator which could be convolved with the dataset schematically illustrated in Fig. 7b (but padded with the extra blank traces as described earlier) in order to achieve the same velocity filtered output. The central peak of the star is the reference point for the convolution and the arms radiate away with a velocity equal to the velocity of the filter cut. The function has planar symmetry in the two planes through the reference point parallel to the time and distance axes and is hence a zero phase operator. As can be seen from Fig. 10, the length of the star arms is proportional to the rate of rejection in the \((\omega-k)\) plane, i.e. inversely proportional to the width of the smoothing zone between the pass and stop bands. However, too rapid a rejection rate may not always be desirable as it can lead to ‘worminess’ in the filtered data.

Application of the Filter

The filter response functions discussed above, i.e. cutting at 1.5 km sec\(^{-1}\) with smoothing over 2, 4, 6 and 8 grid points perpendicular to the line of the cut, were applied to an abbreviated dataset, obtained by taking 64 traces from the shot point trace and padding out to 128 traces with blank columns. The results are shown in Fig. 11a–d where all 128 traces of the array have been plotted out enabling the behaviour of the filter in the appended zero traces to be seen. The extent of the input data is indicated by the black dots. The feature common to all of the outputs is the successful suppression of the ground roll revealing more clearly the flatter lying reflections with an enhanced lateral coherency. However, the dataset treated with the highest rate of rejection (Fig. 11a) displays an unsatisfactory level of worminess caused by the introduction of false events with the apparent velocity of the filter cut. The effect is particularly noticeable in the spill-over of energy from the first arrivals into the blank traces, which in this case wraps round in distance. In order to achieve the high rejection rate, the \((t-x)\) equivalent operator must have long tails, as shown by Fig. 10. When this operator is convolved with the dataset, smearing of energy takes place along the tails, thereby introducing events with dips corresponding to \pm V.

The effect is amply illustrated by applying the velocity filter of Fig. 10 to a sequence of 64 random number traces generated from a Gaussian distribution, depicted before and after filtering in Fig. 12a and b. The Fourier transform of a random real number plane is a half plane of random complex numbers. However, by applying the velocity filter response function to this plane, a significant amount of order has been imposed. Upon transforming back to the time–distance plane, the imposition of the order can be seen in the non-random ‘worms’, the positions of which are determined by local large amplitudes and fortuitous coherencies. This suggests that the post-filtering addition of random numbers to a dataset may minimise the visual impact of the ‘worms’. It is also worth noting that about 30% of the energy has been removed from the random number field.

As the rate of rejection is reduced, with smoothing over 4, 6 and 8 points, the worminess is also reduced, although there is a consequent increasing change in the waveform of those signals lying close to but above the cut-off velocity where the effective bandwidth is reduced. From these trials, it appears that filtering with a
Fig. 11. The results of applying the \((\omega-k)\) transfer functions, from which the \((t-x)\) operators illustrated in Fig. 10 were derived, to a dataset formed by taking 64 traces from the shotpoint in Fig. 6 and adding 32 blank traces on either side of the panel. The extent of the input data is indicated by the black dots. (a) shows the results of the two-point smoothing operator. The rate of rejection is too high resulting in the introduction of false events with the moveout of the cut-off velocity. The energy spillage from the first arrivals is so extensive that it wraps round in distance. (b) is an improvement over (a) with the virtual disappearance of the false event at about 0.75 sec on the left-hand trace. However, the blank traces to the left of the shot point clearly show the spill over of energy which is still just aliased. (c) results from the six-point operator with minimal spill over of energy except at the near shot traces and a level of worminess not significantly worse than that of the unfiltered data. The eight-point operator in (d) produced the shortest tails from the near shot traces. However, it was felt that a slight but noticeable change was taking place in the waveform of the first arrivals, which have a velocity fairly close to the cut-off velocity, due to the smoothing taper cutting into their lobe in the \((\omega-k)\) plane.
Implementation of an Arbitrarily Shaped Filter

The filter transfer functions so far discussed have been restricted to symmetric wedges in the $(\omega-k)$ plane but since there is direct access to the data in the Fourier domain, there is no compelling reason to depend on such simple filters even though they may readily be represented by analytic functions in the $(t-x)$ domain. Since the dataset of Fig. 6 is asymmetric in both domains (Fig. 8a and b) an attempt was made to tailor a filter to the data in the Fourier domain by picking a series of points in the troughs separating the signal lobes A and B from the rest of the field. A cubic spline was fitted to these points representing a function which marked the edge of the stop zone. Rather than taper perpendicularly to the line of the cut, the easier option of tapering in the frequency direction with a $\cos^2$ function was chosen, resulting in the filter transfer function illustrated in Fig. 15.

To go some way towards satisfying the continuity conditions demanded by the fast Fourier transform, the cut-off frequencies at both the positive and negative Nyquist wavenumbers were chosen to be equal, although the first derivatives at these points were not necessarily equal.

As an aside, it is interesting to observe the corresponding star plot, or impulse response function, in Fig. 16. The arms of the star are still straight lines but there are several of them in each quadrant with differing moveouts and offsets from the centre of the star reflecting the fact that the filtering is being performed along an arbitrary dispersion curve as distinct from a non-dispersive phase velocity cut. The filter transfer function and impulse response function are still symmetric in frequency and time, but they are no longer symmetric in wavenumber and offset.

The result of applying the filter to the dataset of Fig. 6 is shown in Fig. 17. Disappointingly, there is little apparent difference between Figs 14 and 17, being the outputs after filtering with the optimum wedge and the tailored transfer function. This was attributed to there being such a good separation between signal and ground roll in the $(\omega-k)$ domain that the dataset was insensitive to the exact line of the cut. It was decided to repeat the exercise with the dataset illustrated in Fig. 18 which was obtained from a shot point only 300 m or so away from the shot point used for the records in Fig. 6.

These records form a split profile with identical sampling in space and time and identical plotting parameters to the dataset of Fig. 6. The dataset transformed into the Fourier domain is illustrated in Fig. 19a and b where it can be seen that although the signal lobe is fairly well defined, there is a great deal of confusion with significant amplitude levels in the rest of the $(\omega-k)$ plane. To isolate the signal lobe from the rest of the field and yet to maintain the cliff face in the negative quadrant of the $(\omega-k)$ plane, it was necessary to pass a cubic spline up the narrow trough with a high rate of rejection so that the filter transfer function would have a fairly sharp edge. The data, after the application of such a mask in the Fourier domain, are shown in Fig. 20, where the integrity of the cliff has almost totally been preserved. The output dataset can be seen in Fig. 21, which shows a great improvement over the input dataset allowing the continuity of shallow reflectors to be traced. The Coal Measures themselves have also been cleaned up with the introduction of very few 'worms'. There is some deterioration in the first arrivals in the negative offset direction but, since a statics scan will have been made by this stage, these arrivals will fall under the mute resulting in no harm to the stack.

Discussion and Conclusions

It has been shown that the $(\omega-k)$ plane is the natural space in which to perform velocity filtering since the
Fig. 13. The amplitude distribution in the $(\omega-k)$ plane after operating upon the dataset shown in Fig. 8 with a cut of 1.5 km sec$^{-1}$ smoothed over 6 points.

Fig. 14. The full 96 trace filtered dataset obtained by transforming back the data illustrated in Fig. 13 into the $(t-x)$ domain.
Fig. 15. The contour plot of the splined filter transform function to be applied to the dataset of Fig. 6 in the \((\omega-k)\) plane. The edge of the stop zone has been chosen to lie in the valleys between the signal and ground roll lobes in Fig. 8b.

Fig. 16. The impulse response function corresponding to the splined transfer function. This contrasts with the star plots of Fig. 10 in having a number of arms with varying moveouts and offsets from the central spike.
Fig. 17. The full 96 trace dataset after filtering using a curved line of cut in the (ω−k) domain. There is, in fact, minimal difference between this output and the filtered output illustrated in Fig. 14 since the curved line passes close to the straight line cut.

Fig. 18 A split profile shot close to the dataset in Fig. 6 with substantial amounts of higher mode ground roll.
Fig. 19. The data of Fig. 18 after transformation into the Fourier domain presented in both projective (a) and contour (b) plots. The signal lobe is apparently well defined by a deep, narrow trough but there is little coherency in the ground roll zone although fairly high amplitude levels exist.
Fig. 20. The data of Fig. 18 after the application of a splined filter with a high rejection rate designed to match the troughs surrounding the signal zone. Note the steep cliff has been retained.

Fig. 21. The output data after transforming back into the time domain. The continuity of shallow events has been improved with the introduction of very little worminess. The reflections from the Coal Measures are clearer with only some deterioration of the first arrivals in the negative offset direction.
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References